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### Semester Two Examination, 2016

### Question/Answer Booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNITS 3 AND 4**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student Number: In figures |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of exam |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 12 | 12 | 100 | 97 | 65 |
|  | | | **Total** | 149 | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

* Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
* Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show** **all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Section One: Calculator-free 35% (52 Marks)

This section has**eight (****8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (5 marks)

The polynomial , where is a real constant, has a zero of . Determine the value of and all other zeros of .

|  |
| --- |
| **Solution** |
| By inspection,    Comparing coefficients,  Comparing coefficients,  Hence  Other zeroes of are and . |
| **Specific behaviours** |
| ✓ deduces another zero of  ✓ uses two zeroes to obtain quadratic factor with real coefficients  ✓ deduces other factor must be of form  ✓ determines value of  ✓ lists required zeroes |

Question 2 (8 marks)

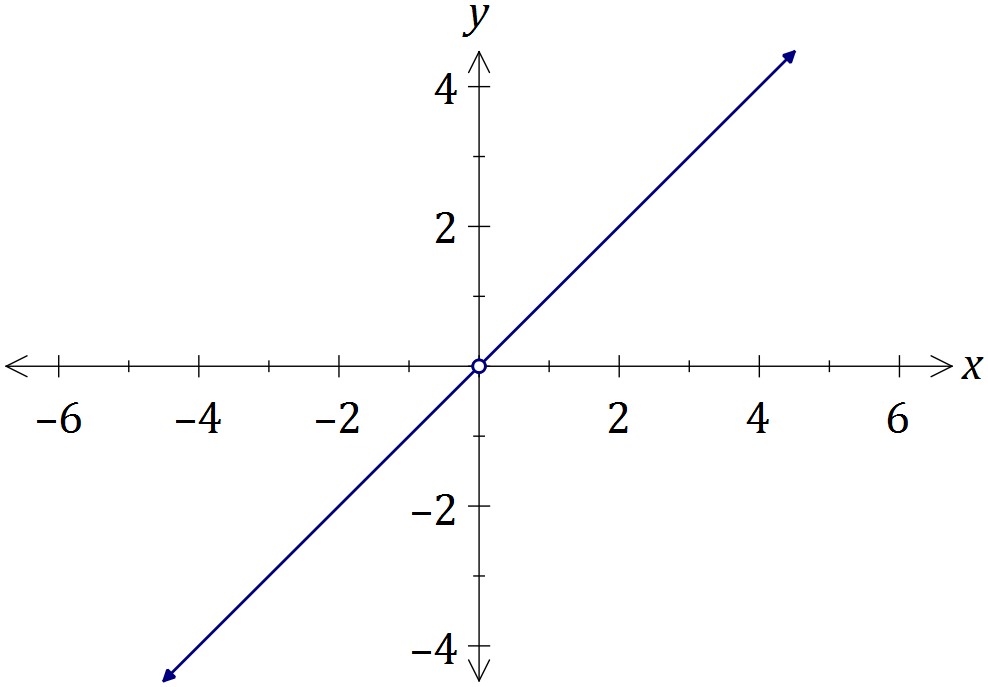
Two functions are defined by and .

(a) Determine the composite function and the domain over which it is defined.

(3 marks)

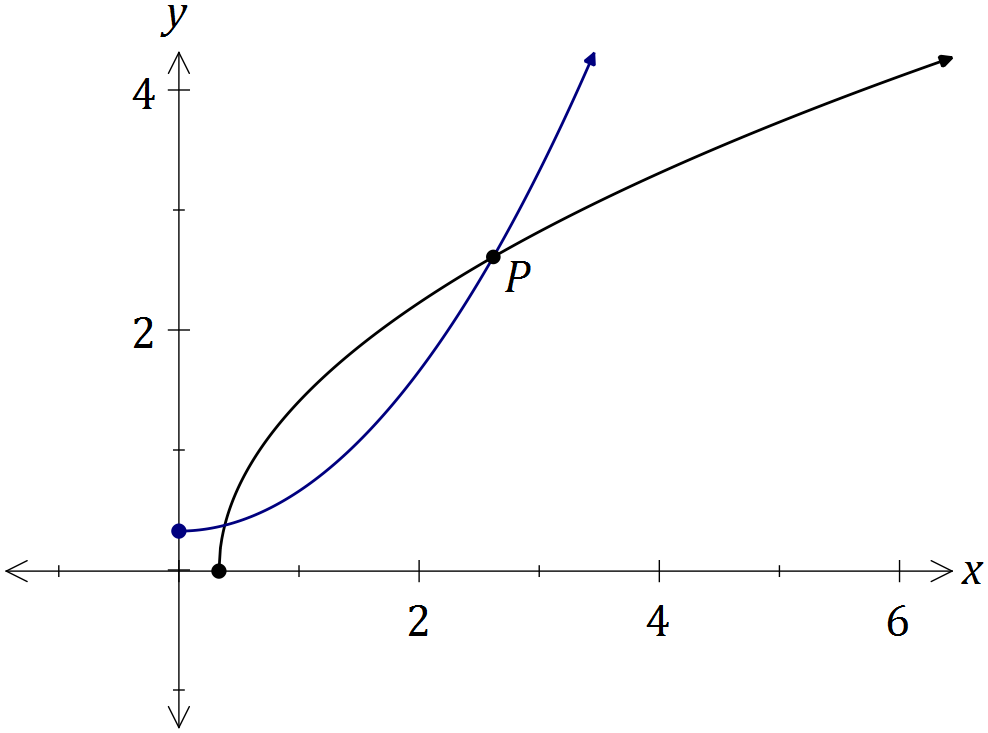
|  |
| --- |
| **Solution** |
| Domain: . |
| **Specific behaviours** |
| ✓ writes composite function  ✓ states upper limit of domain as  ✓ states lower limit of domain as |

(b) Sketch the graph of on the axes below. (2 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ sketches graph of  ✓ indicates discontinuity at |

(c) The graph of is shown below, passing through point with coordinates . Determine , the inverse of , and sketch the graph of on the same axes. (3 marks)



|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines rule for inverse  ✓ sketches inverse using reflection in  ✓ inverse starts at and passes thru |

Question 3 (5 marks)

An object, initially at rest, is dropped from the top of tall building so that after seconds it has velocity meters per second.

The air resistance encountered by the object is proportional to its velocity, so that the velocity satisfies the equation , where is a constant.

(a) Express the velocity of the object in terms of t and k. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ separates variables  ✓ integrates both sides  ✓ evaluates constant a  ✓ expresses velocity as required |

(b) Sensors on the object indicate that its velocity will never exceed 55 metres per second. Determine the value of the constant k. (1 mark)

|  |
| --- |
| **Solution** |
| As |
| **Specific behaviours** |
| ✓ evaluates constant |

Question 4 (5 marks)

Let .

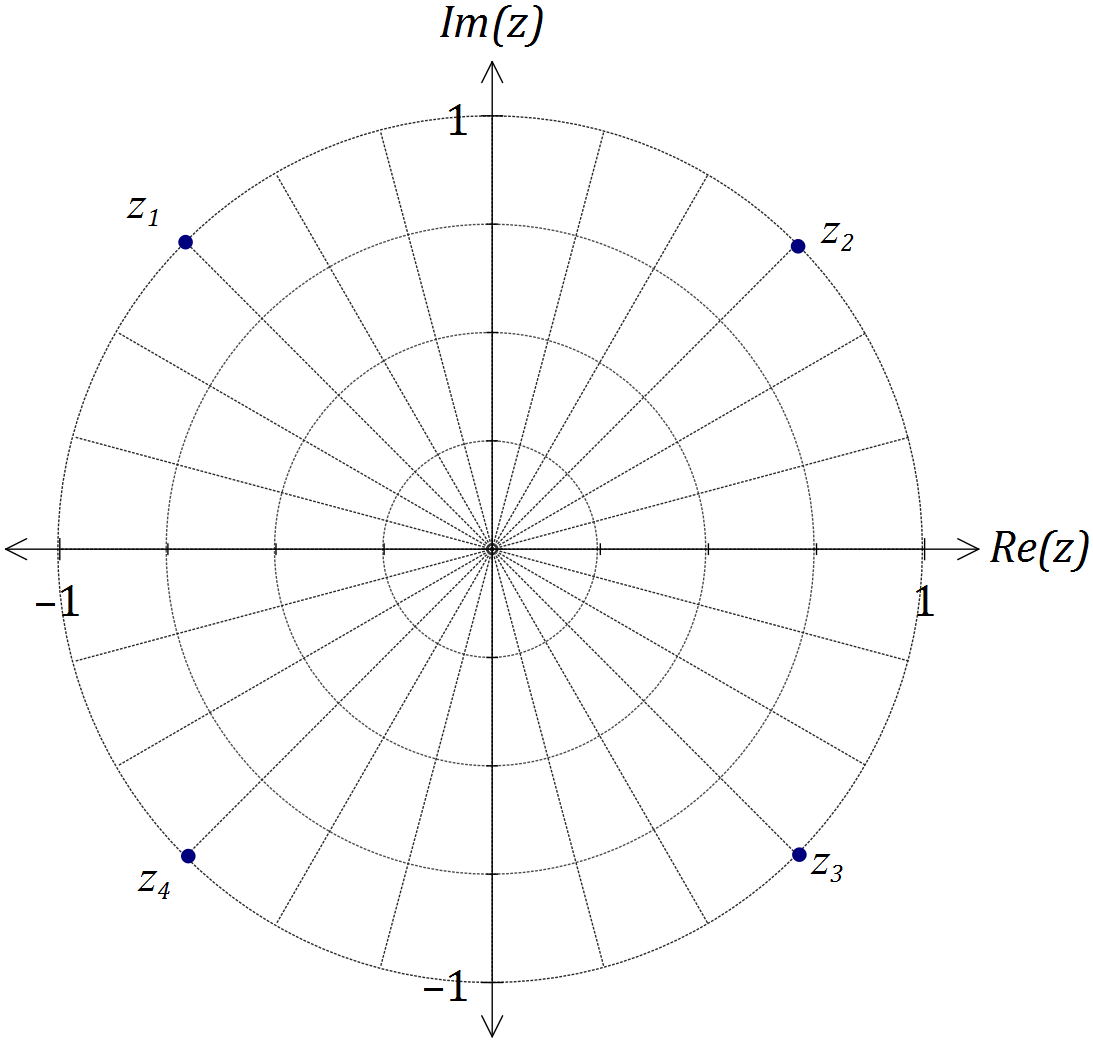
(a) Express in polar form. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines correct modulus  ✓ determines correct argument |

(b) Show that . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states in polar form |

(c) Plot the roots of on the following Argand diagram. (2 marks)



|  |
| --- |
| **Solution** |
| See diagram |
| **Specific behaviours** |
| ✓ adds scale and plots  ✓ plots other three roots |

Question 5 (8 marks)

(a) Using partial fractions, or otherwise, determine . (4 marks)

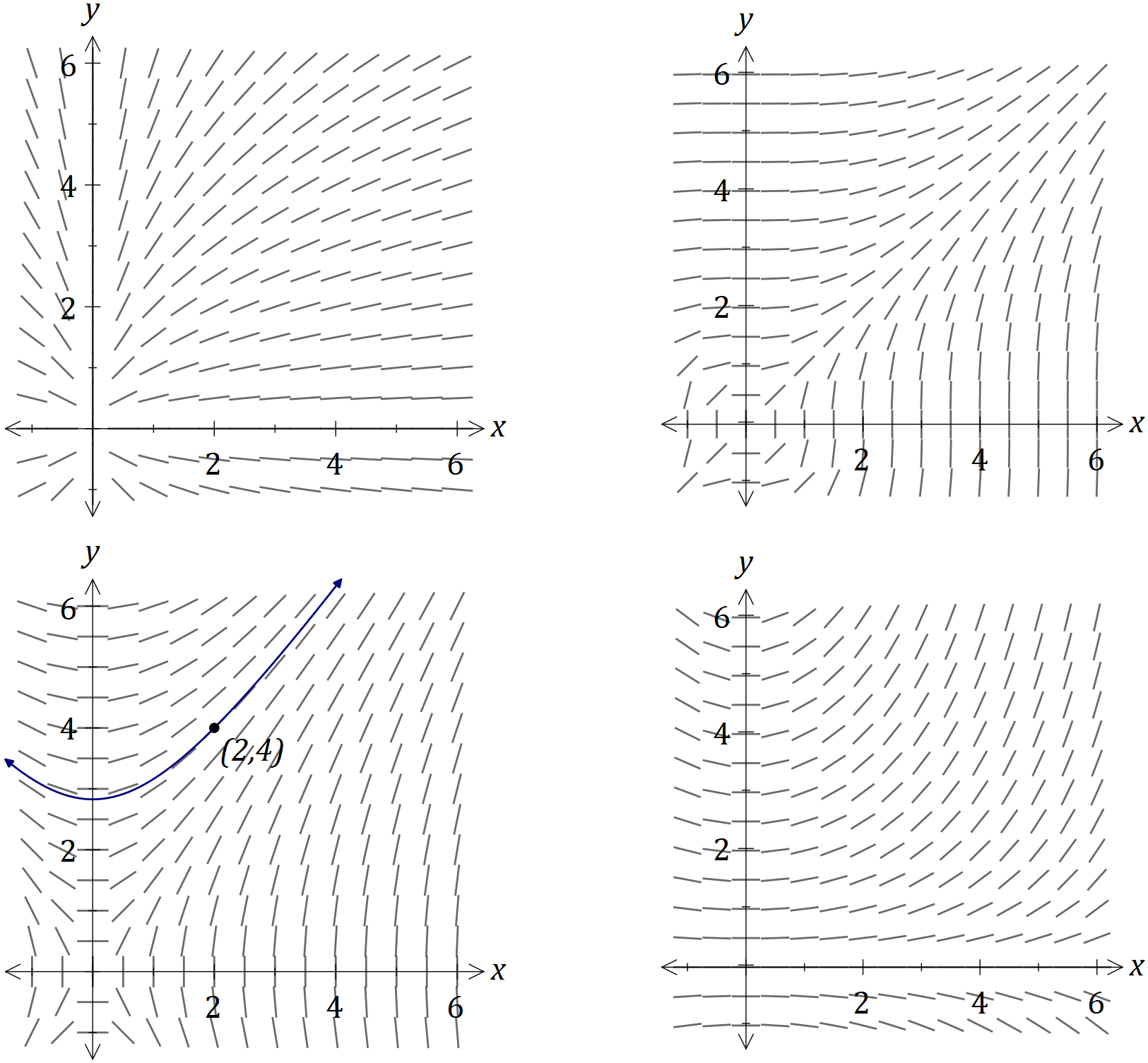
|  |
| --- |
| **Solution** |
| Solving gives |
| **Specific behaviours** |
| ✓ writes equations for A and B  ✓ determines A and B  ✓ integrates both fractions correctly  ✓ includes constant of integration |

(b) Use the substitution to evaluate . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses substitution correctly  ✓ determines new bounds of integration  ✓ integrates correctly  ✓ evaluates integral |

Question 6 (5 marks)

The differential equation is shown in just one of the four slope fields below.



|  |
| --- |
| **Solution** |
| See bottom left-hand slope field |
| **Specific behaviours** |
| ✓ chooses correct slope field  ✓ uses slopes to sketch solution curve right of pt.  ✓ uses slopes to sketch solution curve left of pt. |

(a) On the slope field for , sketch the solution of the differential equation that passes through the point . (3 marks)

(b) Another solution to the differential equation passes through the point Use the incremental formula , with , to estimate the y-coordinate of this curve when . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates change in y  ✓ calculates new y-coordinate |

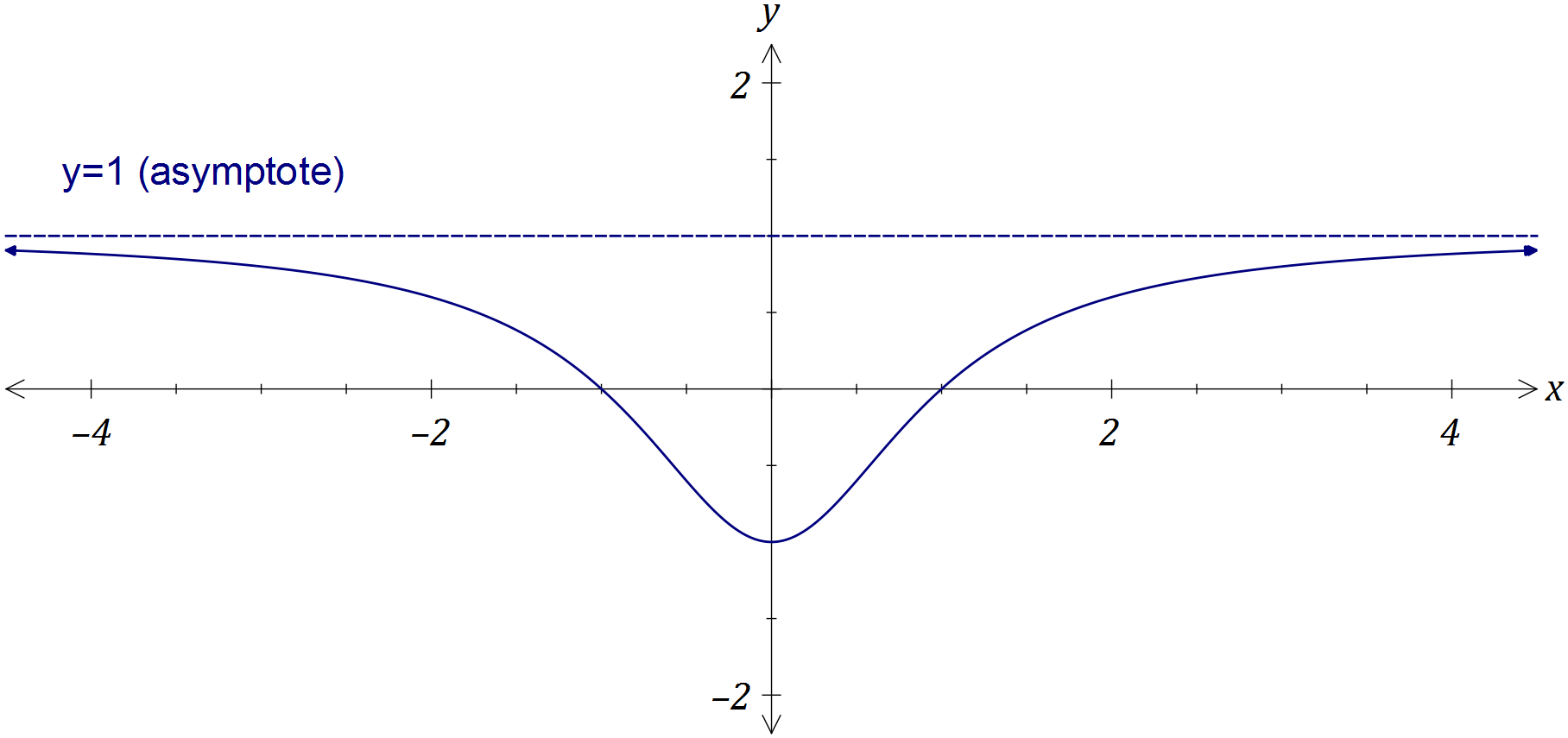
Question 7 (8 marks)

The function is defined as .

(a) Show that the **only** stationary point of the function occurs when . (2 marks)

|  |
| --- |
| **Solution** |
| Hence only when . |
| **Specific behaviours** |
| ✓ differentiates function  ✓ simplifies and makes conclusion |

(b) Sketch the graph of on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ turning point  ✓ roots  ✓ approaches asymptote from below |

(c) Using your graph, or otherwise, determine all solutions to

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states both inequalities |

(ii) . (1 mark)

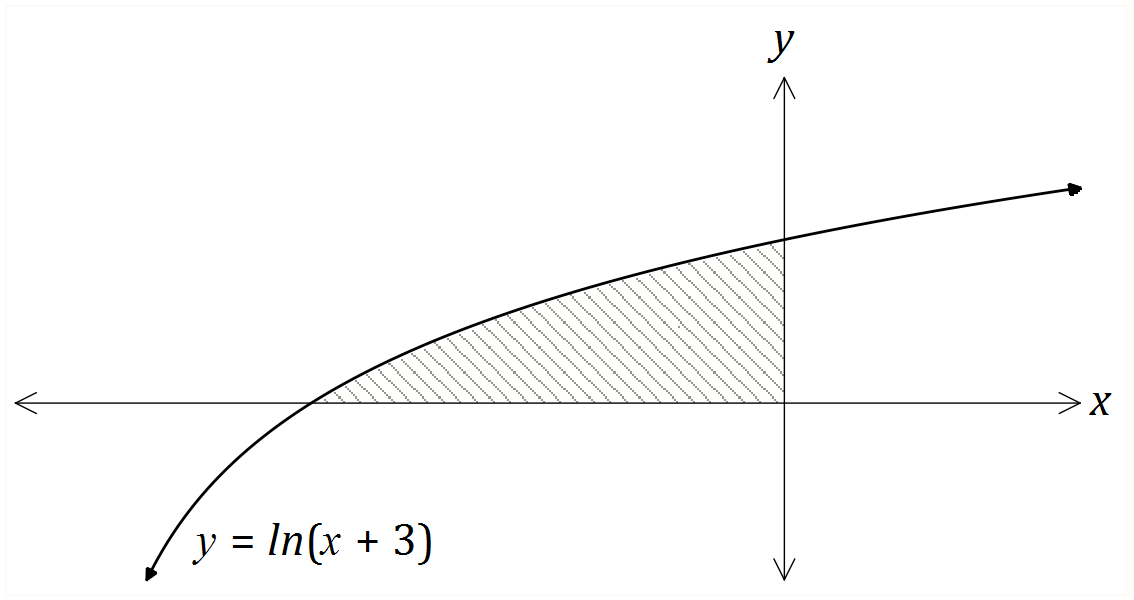
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states all reals |

(iii) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ states solution |

Question 8 (8 marks)

A region is bounded by and as shown in the graph below.



(a) Show that the area of the region is given by . (3 marks)

*(Do not evaluate the integral).*

|  |
| --- |
| **Solution** |
| Region is left of y-axis, so integral will be negative - need to multiply by -1 for area |
| **Specific behaviours** |
| ✓ expresses x in terms of y  ✓ derives upper limit of integration  ✓ explains need to multiply integral by -1, |

(b) Determine the volume of the solid generated when the region is rotated through about the y-axis. (5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct integral  ✓ expands  ✓ integrates correctly  ✓ substitutes upper and lower limits  ✓ simplifies completely |

Additional working space

Question number: \_\_\_\_\_\_\_\_\_

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